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1.0 Purpose

BIAS wants to animate a rack and pinion on its website, and it wants to generate a mathematically correct involute form for the pinion teeth. The purpose of this analysis is to generate a matrix of coordinates that can be read into a Java polygon to be rendered to generate the tooth form.

2.0 Approach

We'll generate a polygon representing half of the tooth form, starting at the center of the root, extending over to the root fillet, stepping up the tooth face to the base circle, tracing the involute, and finishing halfway across the crest of the tooth. We'll let the Java program generate the reflection on the fly. We want to end up with the points on the involute described on a Cartesian coordinate system with its origin at the pinion center and its X-axis passing through the intersection between the pinion pitch circle and the involute tooth face.

3.0 Basic Geometry

BIAS wants to model a 14.5 ($\phi := 14.5\text{-deg}$) degree full-depth involute rack and pinion gear set with a tooth spacing of 10 teeth per inch ($\text{pitch} := 0.10$). According to Ref 1, the smallest pinion that will run without interference with this tooth form has 35 teeth, but we'll use 36 to give an even 10 degrees of arc between the teeth. ($N := 36$), and the resulting angle between teeth is

$\text{pitch_angle} := \frac{\pi}{N}$. Reference 1, (Table 3 Tooth Proportions for Spur Gears, p. 14-06), gives the

following geometry for full-depth 14-1/2 degree involute gears:

The diametral pitch is $DP := \frac{\pi}{\text{pitch}}$, the extension of the tooth beyond the pitch line is

$$\text{addendum} := \frac{1}{DP},$$

and the depth of the tooth form below the pitch line is $\text{dedendum} := \frac{1.157}{DP}$. We'll use the whole

clearance for the root radius $\text{fillet} := \frac{0.157}{DP}$, $\text{fillet} = 4.997 \times 10^{-3}$

$$r_{\text{pitch}} := \text{pitch} \cdot \frac{N}{\pi \cdot 2}$$

$$r_{\text{pitch}} = 0.573$$

$$r_{\text{base}} := r_{\text{pitch}} \cdot \cos(\phi)$$

$$r_{\text{base}} = 0.555$$

$$r_{\text{outside}} := r_{\text{pitch}} + \text{addendum}$$

$$r_{\text{outside}} = 0.605$$

$$r_{\text{root}} := r_{\text{pitch}} - \text{dedendum}$$

$$r_{\text{root}} = 0.536$$

4.0 Involute Calculations

This section details how we calculate the coordinates of points on the involute section. Keeping in mind that we want to end up with a polygon defined on a Cartesian (X, Y) coordinate system centered on the pinion center with the X-axis passing through the intersection between the involute and the pitch circle, we'll begin by building the involute parametrically in a polar (R, T) coordinate system and transform it later. The polar coordinate system will also be centered on the pinion center, but its T=0

axis will pass through the intersection of the involute and the base circle. The involute is the locus of points traced by the end of an inextensible string as it unwinds from the base circle. For a given distance, R, to a point on the involute, the straight length of unwound string is

$S(R) := \sqrt{R^2 - r_{base}^2}$, and the angle measured to the point where the string comes tangent to the base circle is $T_{tangent}(R) := \frac{S(R)}{r_{base}}$. To get the angular distance to the point on the involute,

T(R), we have to subtract off the angle between the tangent radius and the radius vector to the point on the involute $T_{triangle}(R) := \text{atan}\left(\frac{S(R)}{r_{base}}\right)$.

$T(R) := T_{tangent}(R) - T_{triangle}(R)$

Testing, the function:

$T(r_{base}) = 0$

$T(r_{pitch}) = 5.545 \times 10^{-3}$

$T(r_{outside}) = 0.025$

Transforming to the Cartesian coordinates that we want,

$X(R) := R \cdot \cos(T(R) - T(r_{pitch}))$

$Y(R) := R \cdot \sin(T(R) - T(r_{pitch}))$

5.0 Pinion Tooth Polygon

Using the basic tooth geometry from section 3 and the functional description of the involute form from section 4, we can build a polygon that BIAS can plug into its Java program. For now, we'll use a 9 x 2 matrix that represents the root fillet radius as a chamfer and describes the involute using 5 points.

$$\text{pinionPolygon} := \begin{pmatrix} r_{root} & -r_{root} \cdot \tan\left(\frac{\text{pitch_angle}}{2}\right) \\ r_{root} & Y(r_{base}) - \text{fillet} \\ r_{root} + \text{fillet} & Y(r_{base}) \\ r_{base} & Y(r_{base}) \\ \frac{r_{base} + r_{pitch}}{2} & Y\left(\frac{r_{base} + r_{pitch}}{2}\right) \\ r_{pitch} & Y(r_{pitch}) \\ \frac{r_{pitch} + r_{outside}}{2} & Y\left(\frac{r_{pitch} + r_{outside}}{2}\right) \\ r_{outside} & Y(r_{outside}) \\ r_{outside} & r_{outside} \cdot \tan\left(\frac{\text{pitch_angle}}{2}\right) \end{pmatrix}$$

The y matrix just generates a square that can be used to calibrate the output graph.

$$y := \begin{pmatrix} 0.580 & -0.02 \\ 0.580 & 0 \\ 0.600 & 0 \\ 0.600 & -0.02 \\ 0.580 & -0.02 \end{pmatrix}$$

$$ii := 0..8$$

$$iii := 0..4$$

6.0 Rack Tooth Polygon

Set the origin of the Cartesian coordinate system on the intersection of the rack tooth face and the pitch line with the X-axis running along the rack pitch line. Represent one full pitch between tooth crests.

Upper tooth corner

$$x1 := \text{addendum} \cdot \tan(\phi)$$

Upper fillet tangent

$$y2 := -\text{dedendum} + \text{fillet} - \text{fillet} \cdot \sin(\phi)$$

$$x2 := -y2 \cdot \tan(\phi)$$

Lower fillet tangent

$$x3 := x2 + \text{fillet} \cdot \cos(\phi)$$

$$yy := \begin{pmatrix} 0 & 0 \\ 0.02 & 0 \\ 0.02 & 0.02 \\ 0 & 0.02 \\ 0 & 0 \end{pmatrix}$$

$$\text{rackPolygon} := \begin{pmatrix} \frac{\text{pitch}}{4} & \text{addendum} \\ -x1 & \text{addendum} \\ x2 & y2 \\ x3 & -\text{dedendum} \\ \frac{\text{pitch}}{2} - x3 & -\text{dedendum} \\ \frac{\text{pitch}}{2} - x2 & y2 \\ \frac{\text{pitch}}{2} + x1 & \text{addendum} \\ 3 \cdot \frac{\text{pitch}}{4} & \text{addendum} \end{pmatrix}$$

kk := 0.. 4

k := 0.. 7

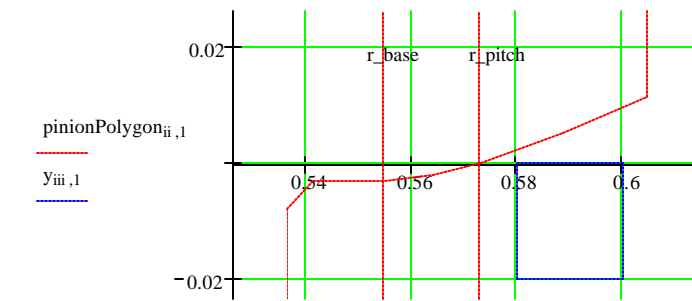
7.0 Results

pitch = 0.1

N = 36

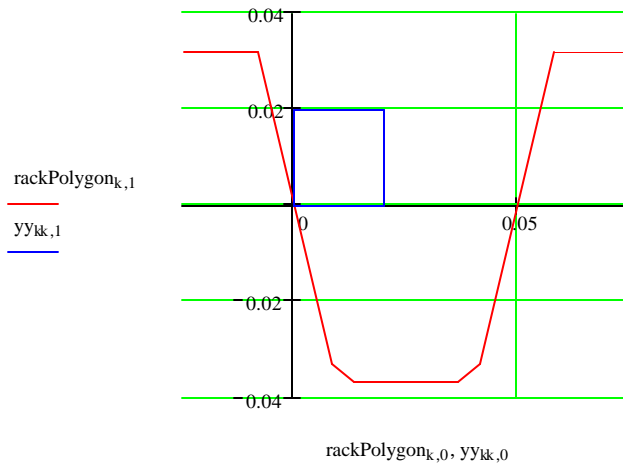
phi = 14.5 deg

7.1 pinion polygon



$$pinionPolygon = \begin{pmatrix} 0.536 & -0.023 \\ 0.536 & -8.073 \times 10^{-3} \\ 0.541 & -3.076 \times 10^{-3} \\ 0.555 & -3.076 \times 10^{-3} \\ 0.564 & -2.013 \times 10^{-3} \\ 0.573 & 0 \\ 0.589 & 4.996 \times 10^{-3} \\ 0.605 & 0.012 \\ 0.605 & 0.026 \end{pmatrix}$$

7.2 rack polygon



$$\text{rackPolygon} = \begin{pmatrix} -0.025 & 0.032 \\ -8.232 \times 10^{-3} & 0.032 \\ 8.556 \times 10^{-3} & -0.033 \\ 0.013 & -0.037 \\ 0.037 & -0.037 \\ 0.041 & -0.033 \\ 0.058 & 0.032 \\ 0.075 & 0.032 \end{pmatrix}$$

References

Carmichael, Ed., Kent's Mechanical Engineers' Handbook, Design and Production, Twelfth Ed., Wiley, 1958.

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file: new_gears.mcd -> new_gears.doc -> new_gears.pdf